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MOTION OF A SPHERICAL CLOUD OF BUBBLES IN A LIQUID WITH MOTIONLESS PACKING

V. V. Dil'man and V. L. Zelenko

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On the basis of Darcy's linear law of resistance, the problem of the ascent of a spherical cloud of bubbles in an infinite liquid with a motionless solid phase is solved. The influence of inertia of the liquid on the character of cloud deformation is discussed.

In investigating the structure of a real bubbling layer with packing under the action of various kinds of perturbation, it is of interest to determine both the distance to which the perturbation of the liquid velocity field excited by a finite region with increased gas content penetrates and the change in this region over time.

In the two-phase case, in the absence of packing, the problem of the collective interaction of bubbles in a cloud was considered in [1], where macroscopic homogeneity of the cloud was assumed, with the consequence that the problem of large-scale liquid motion was not considered, but a new statistical model of the constrained motion of bubbles was proposed. The bubble cloud considered in the present work is macroscopically inhomogeneous, since the gas content is nonuniformly distributed over the liquid-filled space. Therefore, it is necessary to take account of large-scale motion in investigating the hydrodynamic interaction of the phases. In [2], the motion of a macroscopically inhomogeneous cloud of bubbles moving in viscous conditions was considered. The approximate Lamb—Tem method was used in [2]; in this method, in calculating the drag force of the i-th bubble, in the cloud, all the other drag forces are replaced by point forces, when numerical calculation of the combined motion of a few hundred bubbles is possible. However, in the presence of solid phase, no such simple and computationally expedient schematization is possible and, in addition, the bubble motion is usually found to be inertial in character, in practice. On the other hand, if the number of bubbles is sufficiently large, their collective interaction reduces approximately to the interaction of an arbitrarily chosen bubble with the mean velocity field of the liquid arising as a result of the different buoyancies of the elements of the medium, which is uniquely related to the spatial distribution of the gas phase. This approximation may be described by the methods of the mechanics of multivelocity continua based on averaging theory [3]. However, methods of classical filtration theory, which is based on Darcy's linear filtration law, are sufficient for the elucidation of the character of phase motion [4, 5].

A system of phase-continuity and momentum equations is proposed for the description of the phase motion in a three-phase motionless layer; after simple transformations, the system takes the form

$$\partial q/\partial t = \operatorname{div}[(1-q)\bar{V}_1], \tag{1}$$

$$\partial q/\partial t + \operatorname{div}(q\bar{V}_2) = 0, \tag{2}$$

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$$(1-q)(\partial\overline{V}_1/\partial t+(\overline{V}_1\nabla)\overline{V}_1)=-\nabla\Pi/\rho_0-(\nu/K_1)\overline{V}_1+[1-(1-\alpha)q]\overline{g}, \tag{3}$$

$$-\nabla\Pi+(\rho_0\nu/K_2)(\overline{V}_1-\overline{V}_2)=0. \tag{4}$$

The system in Eqs. (1)-(4) is obtained under the assumption of imcompressibility of the phases and an infinitely small gas density. So that the more complex hydrodynamic aspect of the phase interaction may be better elucidated, no account is taken of the scattering of the gas bubbles by the packing, the efficiency of which may be estimated on the basis of simple diffusional representations, from which it follows, in particular, that the role of this mechanism decreases with increase in spatial scale. It is also assumed that Darcy's linear law is satisfied in the phase interactions. The coefficient α describes the part of the Archimedes force acting on the bubble which equilibrates the reaction of the packing. Note that the convective momentum transfer in Eq. (3), of order $\rho_0 V_1^2/R$, may always be neglected in comparison with the filtrational resistance of the packing, which, when $Re_\delta \gg 1$, is determined to a considerable extent by the local pressure gradient, the order of magnitude of which is $\rho_0 V_1^2/\delta$. When $Re_\delta \lesssim 1$, this relation between the terms of Eq. (3) is well satisfied. In addition, the term $\partial\overline{V}_1/\partial t$ may also be significant, since the characteristic time of the nonsteady problem is determined by the relatively fast upward motion of the gas bubbles.

After introducing new scales — the length R, time R/W_0, the velocity W_0, the pressure $\rho_0\nu_0 W_0 R/K_1$, and the gas filling q_0 — Eqs. (1)-(4) are written in dimensionless form, in a coordinate sytem moving at a velocity W_0 along the axis Oz directed vertically upward

$$q_0(\partial q'/\partial t-\partial q'/\partial z)=\operatorname{div}[(1-q_0q')\overline{V}_1'], \tag{5}$$

$$\partial q'/\partial t+\operatorname{div}(q'\overline{\tilde{V}}_2)=0, \tag{6}$$

$$(1-q_0q')S[\partial\overline{V}'/\partial t-\partial\overline{V}_1'/\partial z]=-\nabla p'-\overline{V}'+\mathrm{Ar}q'\overline{e}, \tag{7}$$

$$-m\nabla p'+(\overline{V}_1'-\overline{\tilde{V}}_2)=0. \tag{8}$$

Here $Ar = (1-\alpha)gK_1q_0/\nu W_0$; $S = K_1W_0/R\nu$ are the modified Archimedes and Strouhal numbers; $\overline{V}_1'$ is the liquid velocity in the rest coordinate system; $\overline{\tilde{V}}_2$ is the gas velocity in a moving coordinate system; $p' = (K_1/\rho_0\nu W_0R)(\Pi + \rho_0 gz)$. Consideration is limited to systems with small gas filling $q_0 \ll 1$. The solution of the problem may then be written in the form of a series expansion in terms of the small parameter q_0. The system of equations defining the zero approximation takes the form

$$\operatorname{div}\overline{V}^0=0, \tag{9}$$

$$\partial q^0/\partial t+\operatorname{div}(q^0\overline{\tilde{V}}_2^0)=0, \tag{10}$$

$$S(\partial\overline{V}_1^0/\partial t-\partial\overline{V}_1^0/\partial z)=-\nabla p^0-\overline{V}_1^0+\mathrm{Ar}\,q^0\overline{e}, \tag{11}$$

$$\overline{\tilde{V}}_2=\overline{V}^0-m\nabla p^0. \tag{12}$$

It follows from Eq. (9) that, in the zero approximation, the motion of the liquid phase may be considered as motion of an incompressible homogeneous liquid. This assumption is analogous to the well-known Boussinesq approximation in the theory of heat conduction [7, 8], within the framework of which the change in density of the medium is taken into account only in the term describing the uplift force. In all other respects, the medium is assumed to be incompressible.

After applying the operator div to Eq. (11), it is found that

$$\Delta P^0=\mathrm{Ar}\,\partial q^0/\partial z. \tag{13}$$

Using the equations obtained, the problem of the motion of a bubble cloud initially filling a sphere of radius R is considered. Transforming to dimensionless quantities, the initial gas filling may be written in the form

$$q^0=\chi(\overline{\rho}), \tag{14}$$

where $\chi=\begin{cases}1, & \rho\leqslant 1\\ 0, & \rho>1\end{cases}$ is the characteristic function of a sphere of unit radius.

It follows from Poisson's Eq. (13) that

$$p^0=-(\mathrm{Ar}/4\pi)\int_\Omega[(\partial q^0/\partial z)/|\overline{\rho}-\overline{\rho}'|]\,d\omega,\quad(\overline{\rho}'\in\Omega). \tag{15}$$

Then

$$\partial q^0/\partial z = \bar{e}\cdot \operatorname{grad}\chi(\bar{\rho}) = -\bar{e}\cdot\bar{n}\cdot\delta(\bar{\rho}-\bar{\rho}'),\ \bar{\rho}'\in\Sigma. \tag{16}$$

Substituting Eq. (16) into Eq. (15) gives

$$p^0 = (\mathrm{Ar}/4\pi)\int_{\Sigma}(\bar{e}\,\bar{n}/|\bar{\rho}-\bar{\rho}'|)\,d\sigma. \tag{17}$$

The calculation of the integral in Eq. (17) for the case when Σ is a sphere of unit radius gives the following expressions for p^0: in the gas-filled region, i.e., when $\rho \leqslant 1$

$$p^0 = \frac{1}{3}\mathrm{Ar}\,\rho\cos\Psi, \tag{18}$$

in the external region ($1 < \rho$)

$$p^0 = \frac{1}{3}\mathrm{Ar}\cos\Psi/\rho^2, \tag{19}$$

where ρ is the radius; Ψ is the polar angle of the spherical coordinate system, with its origin at the center of the sphere Σ and its polar axis along the axis Oz. Using Eqs. (18) and (19), the velocity field of the liquid is determined from Eq. (11) under the assumption that the motion is inertialess, i.e., when S = 0

$$\rho\leqslant 1,\ \bar{V}_1^0 = \frac{2}{3}\mathrm{Ar}\,\bar{e}. \tag{20}$$

It follows from the expression obtained that liquid motion in the gas–liquid mixture is quasisolid in character. Substitution of Eqs. (18) and (20) into Eq. (12) gives

$$\bar{V}_2 = [(2-m)/3]\,\mathrm{Ar}\,\bar{e}. \tag{21}$$

Thus, the motion of the gas is also uniform and rectilinear; hence, it follows, in particular, that the spherical bubble cloud is not deformed as it moves. Since $m \lessdot 1$, while $d \leqslant \delta$, the rate of bubble ascent in the cloud exceeds the rate of ascent of a single bubble, according to Eq. (21).

It follows from Eqs. (19) and (11) that, in the external region ($1 < \rho$), the liquid motion is described by the potential of a vertically oriented dipole $\varphi = -m_0\cos\Psi/4\pi\rho^2$ of power $m_0 = (4\pi/3)\mathrm{Ar}$. It is not difficult to establish here [9] that the velocity field of the liquid in this region coincides with the velocity field arising in the motion of a sphere of unit radius (occupying the same volume as the gas–liquid mixture at the given time) in an ideal liquid at a velocity $(2/3)\mathrm{Ar}\bar{e}$.

It follows from the relations obtained that the liquid flow is of potential type both in the gas–liquid mixture ($\rho \leqslant 1$) and in the external region ($1 < \rho$); however, the spherical surface $\Sigma(\rho = 1)$ bounding this region is a vortex sheet [10].

Note that the quasisolid character of the phase motion in the region filled with gas–liquid mixture is a consequence of the spherical bubble cloud. Thus, in the plane case, which corresponds to a cloud in the form of an infinite cylinder with a horizontal generatrix, the liquid motion in the cloud breaks down into two oppositely oriented point eddies; however, this cloud is not deformed as it moves. In the case of a cloud of arbitrary form, even in the inertialess approximation, ascent of the cloud is accompanied by its deformation.

Consider the question of the influence of liquid inertia on the phase motion. It may be shown that, when $S \neq 0$, steady ascent of a spherical cloud of bubbles unaccompanied by its deformation is impossible. It is assumed that such motion exists, and the spherical cloud ascends at some velocity u. Next, Eq. (11) is written in a coordinate system fixed in the bubbles

$$S(-\partial\bar{V}_1^0/\partial t + u\partial\bar{V}_1^0/\partial z) - \bar{V}_1^0 - \nabla p^0 + \mathrm{Ar}\,q^0\bar{e} = 0. \tag{22}$$

Here $\bar{V}_1^0$ is the liquid velocity in the coordinate system fixed in the motionless packing; $\partial\bar{V}_1^0/\partial t = 0$ since steady conditions have been assumed. Limiting consideration to V_{1z}^0 on the vertical axis Oz with its origin at the center of the cloud, and taking account of Eqs. (14), (18), and (19), Eq. (22) takes the form: on the semiline $1 < z$

$$SudV_{1z}^0/dz - V_{1z}^0 = -\frac{2}{3}\mathrm{Ar}/z^3, \tag{23}$$

on the cloud diameter

$$SudV_{1z}^0/dz - V_{1z}^0 = -\frac{2}{3}\mathrm{Ar}. \tag{24}$$

In solving Eqs. (23) and (24), the boundary conditions which must be used are that the liquid is unperturbed at infinity and that V^0_{1z} is continuous at the point z = 1 at the cloud boundary. Denoting the liquid velocity at the point z = −1 by V_- and that at the point z = 1 by V_+, the solution obtained for Eqs. (23) and (24) may be used to write an expression for the difference between these velocities

$$V_- - V_+ = \frac{2}{3}\,\mathrm{Ar}\,[1 - \exp(-2/Su)](1 - I), \tag{25}$$

where

$$I = \int_0^\infty \frac{1}{(Sut+1)^3} \exp(-t)\,dt. \tag{26}$$

It follows from Eqs. (25) and (26) that when Su > 0

$$V_- > V_+. \tag{27}$$

In view of Eqs. (20) and (21), an analogous inequality is valid for the velocity of the gas phase at the given points, and hence steady conditions of cloud ascent are impossible.

Returning to the actual nonsteady motion, consider quasisteady conditions of ascent, in which the initial conditions for the phase velocity coincide with the distribution of the phase velocities in the noninertial approximation already considered. In this case, $\partial \bar{V}^0_i/\partial t \to 0$ when $S \to 0$, since steady conditions are achieved when S = 0. Therefore, when $S \ll 1$, the first term in Eq. (22) may be neglected at times when the deformation of the cloud is small. In this case, Eq. (27), obtained in considering the steady case, will evidently be valid, and hence it follows that the velocity of bubble ascent in the stern region of the cloud is larger than in the frontal region. Thus, in the initial stages of quasisteady ascent of a spherical bubble cloud, the inertial effect consists in flattening of the cloud in the direction of its ascent.

NOTATION

q, q_0, local and characteristic gas filling (volume fraction of gas in gas–liquid mixture); $\bar{V}_1$, $\bar{V}_2$, liquid and gas-phase velocities; ρ_0, true density of liquid; Π, pressure; $\bar{g}$, acceleration due to gravity (vector); K_1, K_2, phase permeabilities of liquid and gas phases; R, radius of bubble cloud; δ, characteristic dimension of the packing element; $Re_\delta = V_1\delta/\gamma$, Reynolds number; W_0, mean velocity of ascent of a single bubble; d, bubble diameter; $\bar{e}$, unit vertical vector; $\bar{\rho}$, radius vector of the point; Ω, gas-filled region; Σ, surface bounding the volume Ω; $\bar{n}$, external normal to Σ; $\delta(\bar{\rho} - \bar{\rho}')$, Dirac delta function; ν, kinematic viscosity of the liquid.

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